THREE STUDIES ON SPECTRAL STRUCTURES OF THE HORIZONTAL ATMOSPHERIC MOTION IN THE TIME DOMAIN

PROEFSCHRIFT

ter verkrijging van de graad van doctor in de wiskunde en natuurwetenschappen aan de Rijksuniversiteit te Utrecht, op gezag van de Rector Magnificus prof. dr. Sj. Groenman, volgens besluit van het College van Dekanen in het openbaar te verdedigen op woensdag 29 oktober 1975 des namiddags te 4.15 uur

door

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geboren op 16 oktober 1947 te Waddinxveen

drukkerij van den dool - sliedrecht

ABSTRACT

The existence and importance of a -3 slope of the spectrum of kinetic energy at sub-synoptic scales are discussed. Estimates of this slope are made, using time series of De Bilt (500 mb) and Cabauw (200 m).

The outcome is that the decrease of kinetic energy with increasing frequency is considerably less than f^{-3}. The exponent is probably closer to -2. The possible causes of this difference are discussed.

1. INTRODUCTION

Power laws play an important role in the theory of three dimensional turbulence. If one assumes that there is a range in the kinetic energy spectrum where no energy is generated or dissipated then it might be that the spectral shape is determined only by the cascade of kinetic energy towards the scales where dissipation becomes important. Therefore such a range is called an inertial subrange. The famous -5/3 law can be derived by assuming that the spectral shape depends only on the rate of dissipation at much higher wavenumbers and on the wavenumber. In inertial subranges local isotropy is believed to occur.

The large scale atmospheric motion is of course also three-dimensional. Nevertheless a simulation with two-dimensional flows has been successful in forecasting the motion. Many features of the large scale atmospheric flow are quasi two-dimensional.

The spectrum of large scale atmospheric motion both in time and space is observed on many occasions. Ranges governed by power laws appear to exist also at these very large scales. If simple relationships exist like

$$E(n) = \alpha n^{-\beta} \tag{1.1}$$

(where E is the spectral energy, n the wavenumber and α and β constants) for the spectra along latitudinal circles, then it might be that the spectral shape can be derived by applying simple dimensional analysis. What quantities can be subject of a cascade and are therefore useful for dimensional analysis. Fjørtoft (1953) gave important indications when he discussed the changes in the spectral distribution of the kinetic energy for two-dimensional non-divergent flow. He derived that both kinetic energy and ½ the relative vorticity squared (=enstrophy) are constant in such a flow. The implication of this, also discussed by Eliassen and Kleinschmidt (1957), is that if an amount of kinetic energy is transferred from an intermediate scale to smaller scales, then necessarily a greater amount of kinetic energy is tranferred from this intermediate scale to larger scales. The energy-containing eddies in a two-dimensional non-divergent flow have the tendency to increase their scale. At the same time enstrophy is transferred mainly to the smaller scales. These tendencies form a well-known feature of, for example, the barotropic model. This model results in an increasing zonality during the motion, which means that energy from intermediate scales is transferred to the largest scales.

In order to maintain flows of energy and enstrophy in the wavenumber space there have to be sources and sinks. In the flow discussed by Fjørtoft, no generation or dissipation takes place; therefore no inertial subrange can be built up. In the real atmosphere kinetic energy and enstrophy are generated mainly at wavenumbers around n=5 by baroclinic processes. Inertial subranges can not exist very close to this input range, so if there is any, it can be expected at (i) about wavenumber 8 and higher, determined by an enstrophy cascade, and (ii) at wavenumbers n=3, 2, 1 by a "reversed" energy cascade. These two inertial subranges have been proposed by Kraichnan (1967) for two-dimensional flow. There have to exist associated sinks of energy and enstrophy. Energy flows to the largest scales which break up in much smaller scales where either a normal cascade towards the scales of dissipation starts or the kinetic energy is converted into potential energy when the vertical stratification is very stable. Enstrophy is cascaded towards higher and higher wavenumbers where it is dissipated by viscosity.

Below all possible power laws, based on inertial transfer of kinetic energy and enstrophy, both for time and space, spectra are given. Such laws are derived from dimensional analysis. Dimensions are given in table 1.

We try to solve now:

$$E(n) = c\ \varepsilon^{\alpha}\ n^{-\beta} \quad \text{and} \quad E(n) = c\ \eta^{\alpha}\ n^{-\beta} \quad \text{and} \qquad (1.2)$$

$$E(f) = c\ \varepsilon^{\alpha}\ f^{-\beta} \quad \text{and} \quad E(f) = c\ \eta^{\alpha}\ f^{-\beta} \qquad (1.3)$$

It was argued by Pasquill (1962) that frequency by itself does not determine a (physical) scale. Therefore we add

$$E(f/\bar{U}) = c\ \varepsilon^{\alpha}\ (f/\bar{U})^{-\beta} \quad \text{and} \quad E(f/\bar{U}) = c\ \eta^{\alpha}\ (f/\bar{U})^{-\beta} \qquad (1.4)$$

Symbol	Description	Dimension
n	wavenumber	L^{-1}
f	frequency	T^{-1}
$E(n)$	kinetic energy contained in wavenumber n	L^3T^{-2}
$E(f)$	kinetic energy contained in frequency f	L^2T^{-1}
ε	"dissipation" of kinetic energy	L^2T^{-3}
η	"dissipation" of enstrophy	T^{-3}
$\bar{U}$	time averaged wind	LT^{-1}

Table 1. A listing of the dimensions of quantities used in order to derive possible power laws.

The solutions of the equations (1.2), (1.3) and (1.4) are listed in table 2.

	space spectrum	time spectrum	
		f	f/U
energy cascade	$E(n)=c\varepsilon^{2/3}n^{-5/3}$	$E(f)=c\varepsilon f^{-2}$	$E(f/\bar{U})=c\varepsilon^{2/3}(f/\bar{U})^{-5/3}$
enstrophy cascade	$E(n)=c\eta^{2/3}n^{-3}$	impossible	$E(f/\bar{U})=c\eta^{2/3}(f/\bar{U})^{-3}$

Table 2. Possible power laws governing the kinetic energy spectrum in time and space.

There is some observational evidence that ranges governed by power laws exist. Julian, Washington, Hembree and Ridley (1970) investigated thoroughly the existence of a -3 range. They found such a range for hemispheric wavenumbers n=8...15. They concluded also that this -3 behaviour was not the product of the smoothing by the analysis. An earlier study by Ogura (1958), using geostrophic approximated wind at 300 mb, reveals that β, see equation (1.2), is the same for both u and v spectra. The values of β increase with latitude, namely from 2.4 at 20°N to 3.5 at 70°N. Ogura concludes that this power is "considerably higher than that predicted from the isotropic turbulence theory (i.e. 5/3)". Ogura also mentions possible dependence of β on the height. Horn and Bryson (1963) made similar investigations at three pressure levels: 300 mb, 500 mb and 700 mb. On the basis of their spatial spectra at 25°, 45° and 65°N, they distinguish two ranges of scales: larger, respectively smaller than about n=5. For the shortest waves they found β = 2.3 to 2.9, while for the larger waves they found a small power $\beta \sim 0.1$. The power β for the smaller scales depends on height: 2.43 at 700 mb, 2.60 at 500 mb and 2.77 at 300 mb. Wiin Nielsen (1967) found for the wavenumber range 8 ... 15 a power around β = 3 in the free atmosphere. He also mentions the pressure dependence: 200 mb: 2.8; 300 mb: 3.1; 500 mb: 2.9; 700 mb: 2.6; 850 mb: 2.4; 1000 mb: 1.9. These results were derived from maps of 8 pressure levels,analysed during one year.

The -5/3 law in connection with a reversed cascade at the largest possible scales is never clearly reported. The kinetic energy decreases undoubtedly with increasing wavenumber but the exponent β is less than 5/3. This can be seen for example, from figure 4 in Part I of this thesis. The large scale motion is strongly anisotropic at these scales.

Numerical simulations of a purely two dimensional flow support the prediction of a -3 slope (Lilly, 1969). Also the simulation of the general circulation as reported by Manabe et al (1970) indicates that a range with a power of about -3 possibly exists.

The -3 and -5/3 slopes of the kinetic energy spectrum are expected on the basis of dimensional analysis using conserved and cascaded quantities. This is one approach to the problem but not the only approach possible. First of all also other quantities than those that are subject to a cascade, can be used for dimensional analysis if a sound physical basis can be found to do so. No example of such an approach is known to the author. Secondly it can not be proved that (only) dimensional analysis leads to the correct solution of the problem. Without the use of dimensional analysis power laws can be derived as well if certain assumptions are made. An example can be found in a paper by Phillips (1971), where a -2 slope in the kinetic energy spectrum in both the frequency and the wavenumber domain is derived. The basical assumption of Phillips was that the variance of the considered quantity, in our case u or v, is mainly contributed to by (almost) discontinuous changes.

A -2 law has been derived also by using completely different arguments. In studies of three dimensional turbulence the autocorrelation function, being defined as

$$g(\tau) = \overline{u_i(t)\ u_i(t+\tau)}/\overline{u_i^2(t)} \quad \text{(where } \tau \text{ is the time lag)}$$

has been approximated sometimes by (see Hinze, 1959):

$$g(\tau) \sim e^{-\tau} .$$

Remarkably this yields also a -2 power law at high frequencies:

$$\Phi_{u_i u_i}(f) \sim (1+f^2)^{-1}$$

An objection against the existence of a -3 law may be that the large-scale atmospheric flow is not exactly two dimensional. Charney (1971) showed, however, that for quasi-geostrophic flow the -3 law of two dimensional flow is valid. In addition to the two horizontal wind components then also the eddy available potential energy obeys a -3 law in the range where enstrophy is cascaded.

An inertial subrange develops only if the sources and sinks in the spectrum do not overlap. Therefore it is favourable if the source is limited to a narrow range of scales. In the range where a -3 exponent is expected no kinetic energy is believed to be produced. But Steinberg (1972) made computations which indicated that in that particular part of the spectrum the conversion of eddy available potential energy into eddy kinetic energy is considerable. Also Dutton and Johnson (1967) found that this conversion is not restricted to a narrow spectral range near n=5. An input of kinetic energy may change the character of the pure enstrophy cascading range.

These latter facts indicate that although there is observational evidence concerning the existence of a -3 range, the character of this part of the spectrum is not at all certain.

Knowledge concerning the spectral shape near the high frequency side of the synoptic scales is important because the predictability of the atmosphere depends critically on this spectral shape. In a model, describing the atmosphere as well as possible, inevitable errors in the initial state play a role. These errors can be decreased by making the distance between aerological stations smaller, but this does not necessarily mean that the range of predictability is extended. Moreover the error can not be made as small as desired for practical reasons. Lorenz (1969) and Leith (1971) have studied how this "error energy" influences the prediction. They assume that in the initial state, error energy is associated with scales comparable with the grid distance. During the run of the model this error energy grows and propagates towards larger scales to arrive finally at the largest scales to be predicted. The limit of predictability is defined in such a way that predictions based on initial states differing only in the smallest resolvable scales, differ as much as predictions based on randomly chosen initial states. This limit depends on the spectral shape as can be seen from the following reasoning (Lorenz, 1969 and Lilly, 1973). An uncertainty at wavenumber 2n propagates towards wavenumber n in a time $\tau(n)$, for dimensional reasons assumed to be proportional with n^{-1} and $v(n)^{-1}$, a characteristic velocity

$$\tau(n) \cong n^{-1}\ v^{-1}(n) \tag{1.5}$$

$v(n)$ can be expressed in $E(n)$ as

$$v(n) \cong n^{\frac{1}{2}}\ E^{\frac{1}{2}}(n)$$

therefore,

$$\tau(n) \cong n^{-3/2}\ E^{-\frac{1}{2}}(n)\quad . \tag{1.6}$$

Now suppose the spectral form of $E(n) = c\ .\ n^{-\beta}$, then

$$\tau(n) \cong n^{\beta/2-3/2}\quad . \tag{1.7}$$

Therefore the larger the negative power in the spectrum the more time it takes for initial errors to influence the scales to be predicted.

The time needed for an uncertainty at scale $2^N n$ to infect the scale n is given by

$$\tau(2^N n \to n) \cong \tau(n) \sum_{l=0}^{N-1} 2^{l(\beta-3)/2} \tag{1.8}$$

The value of N is determined by the density of the observational network. For example, at present the initial states used in numerical models contain reliable information for wavenumbers up to n=15 or 20 only. Therefore $N \approx 4$ to 5. The principle question to be posed here is whether the series in (1.8) diverges if $N \to \infty$, that is when the density of the observational network goes to infinity. In that case the series becomes divergent if the power $(-\beta)$ is smaller than or equal to -3. $\tau(2^N n \to n)$ may reach very large values in that case. Therefore it is of extreme importance to know the power in the actual atmosphere. If the power is larger than -3 the range of predictability is limited whatever the density of the observational network is.

Such conclusions can only be drawn on the basis of equation (1.5). The validity of this equation, however, should still be a point of discussion.

The investigation of the spectral behaviour at sub-synoptic scales is difficult in practice. The enstrophy cascade process is formulated in terms of space scales. Therefore it is natural to extract information from analysed weathermaps. However, those maps only give reliable information up to scales n=15 to 20. Only in models of the atmosphere this range can be extended further. To the knowledge of the author at present no experiments are in preparation with aerological stations situated along a latitudinal circle, and separated by much less than a few hundred kilometers. There is no direct knowledge of the space spectrum for n>15, therefore.

Some light can be thrown on the smaller scales indirectly by using time series. Measurements done at fixed points with small sampling distances covering periods of months or even years are available. Such time series contain valuable information concerning the high frequencies corresponding to spatial scales n>15. However, only time spectra can be determined and a "transformation" to the space domain is necessary. We have to accept this difficulty because time series appear to be the best information we have at present.

2. DATA AND ANALYSING METHOD

Two time series were chosen in order to make computations of the slope of the kinetic energy time spectrum. A time series of the wind in De Bilt at 500 mb (data set I) was extracted from the yearbooks of the Royal Netherlands Meteorological Institute. The sample distance Δt is 6 hours, the times of measurement being 00.00, 06.00, 12.00 and 18.00 G.M.T., whereas the record length m is two years: 1966 and 1967. Of course a resolution at higher frequencies would be very desirable but data set I permits only resolution up to frequencies of 1 cycle per $2\Delta t$ = 1 cycle per 12 hour. No time series concerning the free atmosphere with smaller sample distances were available. In order to overcome this shortcoming a second data set (II) consisting of measurements at the meteorological tower in Cabauw was used. At the height of 200 meters 48 ½-hour averages per day of the wind speed and wind direction were available during the summer period 22nd June, 1973 to 8th August, 1973. By using this series a resolution up to f=1 cycle per hour can be reached.

One has to be very careful in comparing the results of computations based on data sets I and II, however. The height at which these series were registered is quite different and the vicinity of the earth's surface may influence the spectral properties determined with the aid of data set II. Furthermore, the effect of aliasing is probably quite different for both series. Spectral energy associated with frequencies that can not be resolved (because Δt is too large) is negligible in the case of time series II. Data set I consists of "instantaneously" observed real winds. Such instantaneous observations are in reality averages over layers of one hundred or more meters thickness. Nevertheless, such observations might be influenced by frequencies much higher than the highest resolvable frequency of 1 cycle per 12 hours. In practice aliased energy is assumed to be mainly present at the highest resolved frequencies (Oort and Taylor, 1969).

Both series or parts of these series were analysed as follows.

1. The variances $\overline{u'^2}$, $\overline{v'^2}$ and the covariance $\overline{u'v'}$ are computed, u and v being the west-east and south-north components of the horizontal wind respectively, $\bar{u}$ and $\bar{v}$ are the record averages and u' and v' are the deviations of these averages: $u'(t) \equiv u(t) - \bar{u}$.

2. The record averages and the linear trends (estimated by the least squares method), are subtracted from the u and v time series. This yields u^* and v^* time series.
3. The new, u^*, v^* series are developed in Fourier series using the fast Fourier transform

$$u^*(t) = \sum_{f=1}^{M} a(f)\cos ft + b(f)\sin ft$$

$$v^*(t) = \sum_{f=1}^{M} c(f)\cos ft + d(f)\sin ft$$

where a, b, c and d are Fourier coefficients, M is the number of terms in the development, f is the frequency,measured in cycles per recordlength m and t is time, measured from 0 to 2π.
4. Variance- and covariance spectra $\Phi_{u_i u_j}(f)$ are now determined by:

$$\Phi_{uu}(f) = (a^2(f) + b^2(f))/2$$

$$\Phi_{vv}(f) = (c^2(f) + d^2(f))/2$$

$$\Phi_{uv}(f) = (a(f)\,c(f) + b(f)\,d(f))/2$$

The contribution of trends and the time mean flow being excluded, the kinetic energy of the horizontal motion (per unit mass) can be written as

$$(u^{*2} + v^{*2})/2 = \sum_{f=1}^{M} (\Phi_{uu}(f) + \Phi_{vv}(f))/2 \quad (\equiv \sum_{f=1}^{M} E(f))$$

The covariance spectrum $\Phi_{uv}(f)$ is not directly important in determining the slope of the kinetic energy spectra but gives information whether or not isotropy really exists in the considered range of frequencies. In an isotropic range the correlation between u and v has to be very small by definition.

3. RESULTS AND DISCUSSIONS

With data sets I and II we have a possibility to compute spectra at frequencies higher than those corresponding to the synoptic scale. Figure 1 shows the graphs of $\Phi_{uu}(f)$ and $\Phi_{vv}(f)$ drawn for f = 180 cycles per 2 years and higher. For reference a -3, -2 and -5/3 slope are drawn too.

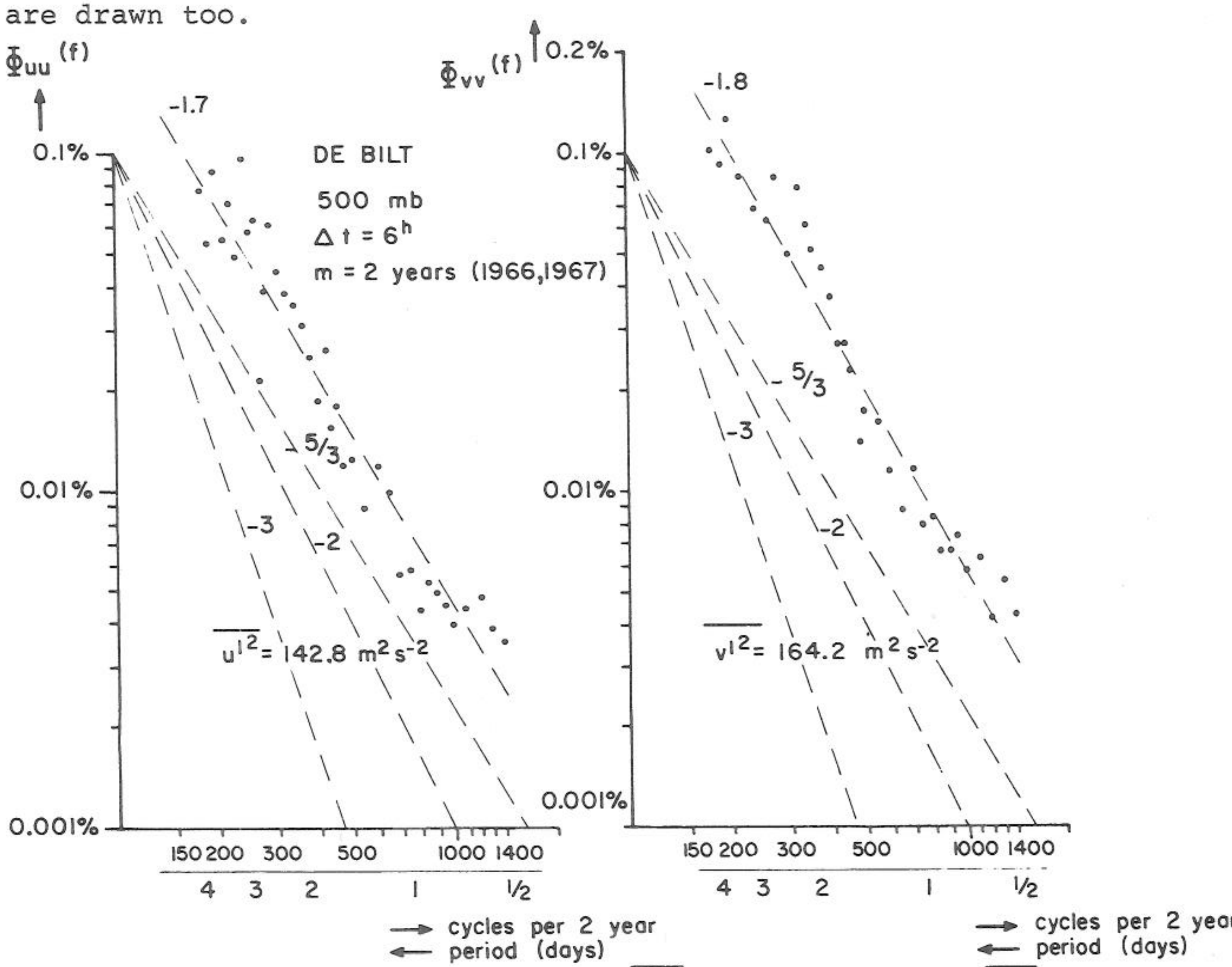

Figure 1. Spectral distribution of $\overline{u'^2}$ (left part) and $\overline{v'^2}$ (right part) at the high frequency side of the synoptic scale. The abcissa gives the frequency in cycles per two years and the period in days whereas the ordinate gives the contributed variance per harmonic as a percentage of the total variance. Dots in the graph represent non-overlapping averages over individual estimates of $\Phi_{u_i u_i}(f)$.

On a logΦ/logf presentation these powers appear as straight lines. As can be seen the slope from the frequency of 1 cycle per 4 days towards higher frequencies is much less than $|-3|$. The value of the slope was estimated with least squares and amounted to -1.7 and -1.8 for the u and v spectrum respectively. It is encouraging that the u and v component give comparable results. This supports the theory that local isotropy occurs in this range. In figure 2 a plot of $\Phi_{uv}(f)$ is given.

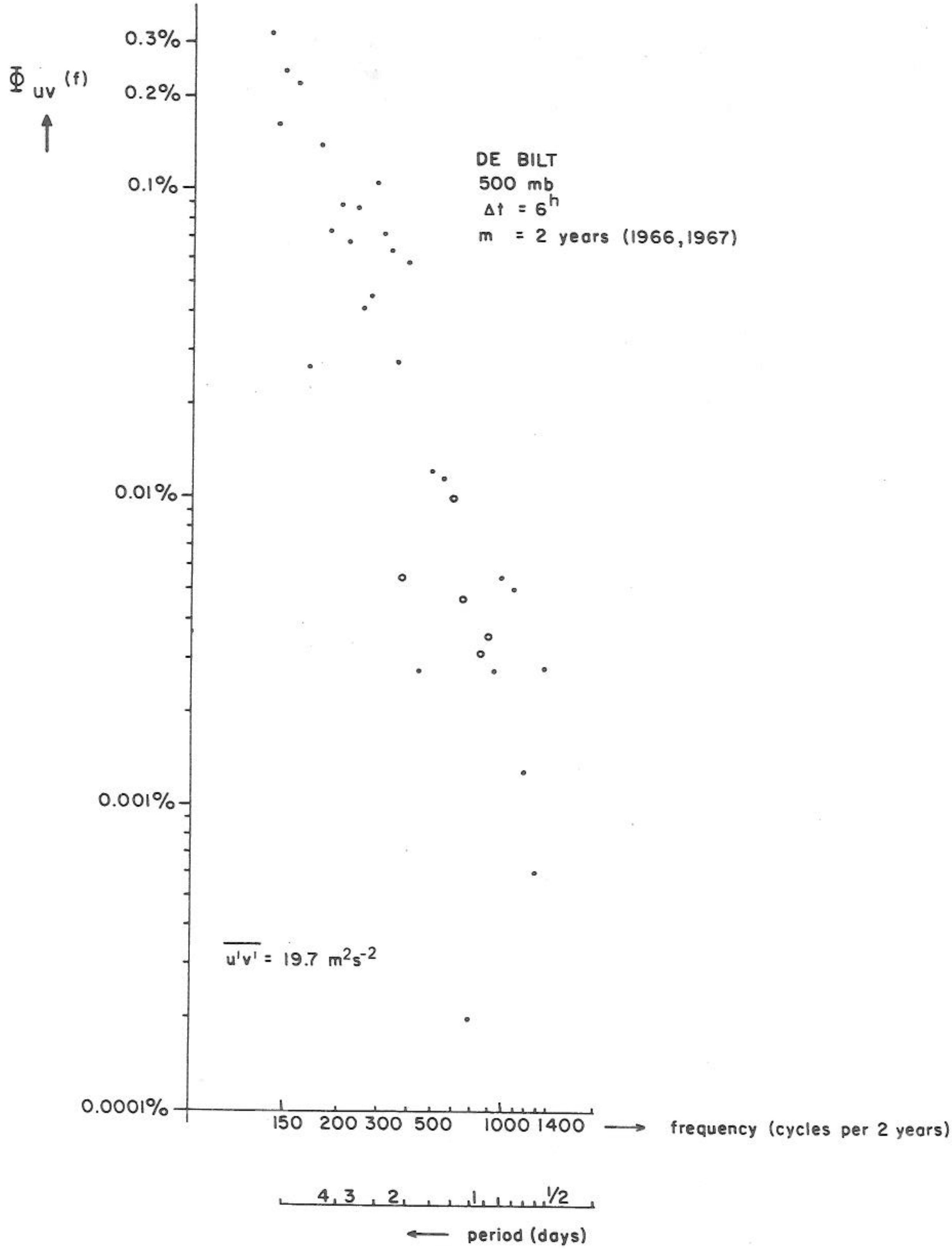

Figure 2. Spectral distribution of $\overline{u'v'}$ at the high frequency side of the synoptic scale. The abcissa gives the frequency in cycles per two years and the period in days whereas the ordinate gives the contributed covariance per harmonic as a percentage of the total covariance. Dots in the graph represent non-overlapping averages over individual estimates of $\Phi_{u_iu_j}(f)$. Open dots indicate negative contributions.

This informs us concerning the correlation or coherence in the frequency range considered. The decrease of $\Phi_{uv}(f)$ with increasing f is much faster than the decrease of $\Phi_{uu}(f)$ and $\Phi_{vv}(f)$ with f. Obviously the conditions become more and more isotropic with increasing f.

In inspecting figure 1, one gets the impression that a slight change in the slope occurs at about 1 cycle per day in the graphs of both the u and v component. At lower frequencies the slope seems to be somewhat steeper.

Data set II makes it possible to continue the spectra of figure 1 towards higher frequencies. One has to realize however, that both heights (500 mb, 200 m) and periods of measurements (1966/1967, summer 1973) are different. Nevertheless the similarity of the spectrum for Cabauw, figure 3, and the spectrum for De Bilt is striking.

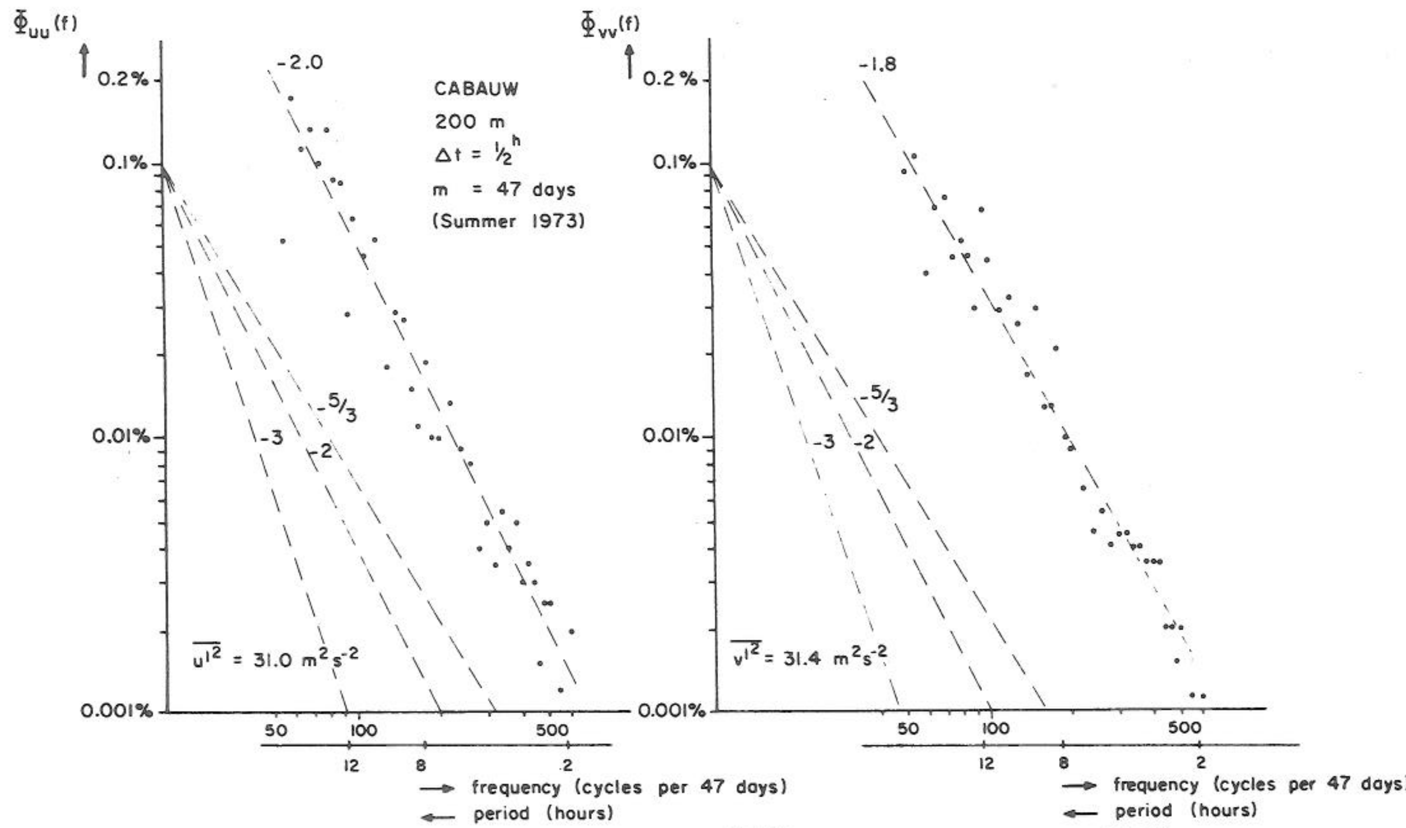

Figure 3. Spectral distribution of $\overline{u'^2}$ (left part) and $\overline{v'^2}$ (right part) at high frequencies. The abcissa gives the frequency in cycles per 47 days and the period in hours whereas the ordinate gives the contributed variance per harmonic expressed as a percentage of the total variance. Dots in the graph represent non-overlapping averages over individual estimates of $\Phi_{u_i u_j}(f)$.

The slope between -5/3 and -2 continues into the mesoscale range where only very little energy appears to be present. This part of the spectrum is often referred to as a gap (see Fiedler and Panofsky, 1970). Again the u and v spectrum show approximately the same slope. According to a least squares fit these slopes are -2.0(u) and -1.8 (v).

Figure 4 gives a plot of $\Phi_{uv}(f)$. It can be seen that $\Phi_{uv}(f)$ decreases more rapidly than $\Phi_{uu}(f)$ and $\Phi_{vv}(f)$ do when f increases. Therefore also in the Cabauw sample isotropic conditions become prevailing at higher frequencies.

From our results it appears that a slope between -5/3 and -2 fits very well the kinetic energy spectrum over a relatively broad range. This conflicts seriously with the findings of Julian et al (1970), Wiin Nielsen (1967) and others who found rather a -3 slope. Until now "conflicting evidence", where the existence of a -3 slope is concerned (GARP, 1973), was found by Vinnichenko (1970). Vinnichenko found in the time spectrum a power of approximately -5/3 where -3 is expected. Now -1.7, -1.8, -2.0 and -1.8 are very close to -5/3 so that our results support much more the findings of Vinnichenko than those of Wiin Nielsen etc.

In addition to the paper by Vinnichenko also a paper by Kao (1965) has to be mentioned. Kao computed $\Phi_{uu}(f)$ and $\Phi_{vv}(f)$ from measurements every 6 hours at the 300 mb level over Salt Lake City during the International Geophysical Year 1958. Kao claims that the kinetic energy spectrum is proportional to f^{-2} in the high frequencies.

It has been stated by Charney (1971) that in addition to the kinetic energy also the spectrum of available potential energy obeys a -3 law. A time spectrum of daily values of the available potential energy for the northern hemisphere cross-section along 75^{o}W for 1958 revealed a -2 decrease at high frequencies (Dutton and Johnson, 1967). This is consistent with our findings concerning the spectral behaviour of the kinetic energy in the frequency domain.

The question arises why we did not find a -3 slope. It is remarkable that time spectra (Vinnichenko, Kao, Dutton et al) give much gentler slopes than space spectra do, see Julian, Wiin Nielsen etc. In principle the differences can be explained in two ways.

1. The slopes derived from time spectra are due to sampling errors different from those obtained in the space domain. Probably this is no good explanation.
2. A second explanation may be that time and space series inevitably contain different information. It can be asked whether series, measured at a fixed point every Δt, are comparable with series either measured along a latitudinal circle, or series read out of analysed maps. A very important phenomenon at midlatitudes is formed by the disturbances in the polar front. The spatial dimensions of such disturbances are in the beginning a few hundred kilometers. Therefore they can not be detected very well in analysed isohypse fields like the one used by Ogura, Horn and Bryson, Wiin Nielsen and Julian. Nevertheless these systems of limited size sometimes contain considerable amounts of kinetic energy at scales where the -3 slope is supposed to be present. On the other hand, at fixed points events like the passage of a frontal disturbance can be registered very well. The conclusion therefore is that the role of smaller scales of motion is appreciated quite differently in time and space series. The occurrence of fronts gives rise to slopes gentler than -3. Phillips (1971) showed that a decrease of the kinetic energy spectrum according to f^{-2} is to be expected if almost discontinuous changes in the velocity components explain the greater part of the variance of the wind components. Charney (1973) mentions the possibility that the kinetic energy falls off with decreasing scale of motion first with a -3, then with a -2 and finally with a -5/3 slope. Maybe the height-dependence of the power, ranging from -3 at 200 - 300 mb to -1.7 at 1000 mb, as found by Wiin Nielsen (1967) can be interpreted in this connection as the height dependence of the kinetic energy associated with frontal motion. This suggests that also space spectra give a decrease of kinetic energy with an exponent closer to -2 if the frontal waves are sufficiently taken into account.

Even when space and time samples contain the same information, the link between both remains difficult. Can we compare time and space spectra?

In table 2, $f/\bar{U}$ was introduced as a "scale" rather than f itself. Dimensionally this is correct but it is not certain that we can estimate scales with the help of measurements made at one point. If however, $n = f/\bar{U}$ is accepted, a plot of E(n) against n is only a parallel displacement of E(f) against f on a logE/logf paper. Therefore the same power law is to be expected for both the frequency and wavenumber domain, although direct dimensional analysis using E(f), f and η gives no solution at all.

4. CONCLUSIONS

- The spectrum of kinetic energy decreases approximately as $f^{-\beta}$ at the high frequencies.
- For De Bilt, 500 mb, $\beta \approx 1.7$ to 1.8
- For Cabauw, 200 m, $\beta \approx 1.8$ to 2.0
- The slopes are considerably gentler than the predicted value of -3. The -3 slope is found on several occasions but always in the wavenumber domain. The time spectra available give a slope comparable to the one found in this study.
- The difference between time and space spectra may be explained from the fact that time and space samples contain different information. Especially the appreciation of frontal motion may be important in this connection.

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